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A Unique Solution for Flight Body Angular Histories

by Thomas E. Harkins

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A Unique Solution for Flight Body Angular Histories

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14. ABSTRACT

U.S. Patent 638155, Projectile Orientation In Navigation TERms (POINTER), was awarded in June 2002 for the design of a constellation of optical and magnetic devices and a processing methodology that provides a solution for the projectile pitch and yaw angular history. The POINTER solution is obtained from the simultaneous solution of a system of equations describing respective sensor output as functions of projectile orientation. Because one of these equations is quadratic, there are two possible solutions. An alternate set of simultaneous equations that yield a unique POINTER solution have been identified and is presented herein.

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1. Introduction

For many years, the Advanced Munitions Concepts Branch of the U.S. Army Research Laboratory's (ARL's) Weapons and Materials Research Directorate has been producing trajectory heading histories for flight experiments of developmental projectiles using data telemetered from strap-down optical and magnetic sensors (Hepner & Harkins, 2001). U.S. Patent 638155, Projectile Orientation In Navigation TERms (POINTER), was awarded in June 2002 for the design of a constellation of optical and magnetic devices and a processing methodology that provides a solution for the projectile pitch and yaw angular history. The POINTER solution is obtained from the simultaneous solution of a system of equations describing respective sensor output as functions of projectile orientation. Because one of these equations is quadratic, there are two possible solutions. This ambiguity usually is easily resolvable by the analyst when s/he is post-flight processing the data. However, automated and on-board processing sometimes cannot identify the correct solution. An alternate set of simultaneous equations that yields a unique POINTER solution has been identified and is presented herein.

The "legacy" POINTER solution is reviewed in sections 1 and 2 before we proceed to the development of the new POINTER solution in sections 3 and 4.

2. Solving for Projectile Heading: Generalized Solution

The center of motion and the principal axis of rotation, i.e., the spin axis, of a three-dimensional freely flying solid body are described within a Cartesian coordinate system with the use of three variables denoting location and two variables defining angular orientation. The derivative of the location with respect to time is commonly referred to as the velocity vector, $\vec{V}$. For a symmetric, spinning body, the navigation pointing vector, $\vec{P}$, normally is coincidental with the principal axis of rotation. The rotation rate of the body about this axis is commonly called the "spin rate". The Eulerian heading variables psi (ψ) and theta (θ) are used to denote the two angular components of azimuth and elevation required to orient the body principal axis of rotation within a right-handed Cartesian navigation system. The principal axis of rotation of a spinning body is often not co-linear with the velocity vector. In such cases, an orientation time history estimated from derivatives of location variables does not provide an accurate measure of the navigation pointing angles of the flight body. With the combination of body-fixed sensors' measurements of the included angles between the axis of rotation and two distinct earth-fixed fields and knowledge of these fields' orientations in the navigation coordinate system, ψ and θ can be determined. Thus, a pointing angle time history can be generated.

The parameters just described are all seen in figure 1 where a projectile with its pointing and velocity vectors is shown in an exemplary right-handed coordinate system along with two representative field vectors of known orientation, $\vec{F}_1$, and $\vec{F}_2$. The included angles between the spin axis and the two field vectors (σ_1 and σ_2) and the azimuth (ψ) and elevation (θ) heading angles are also shown.

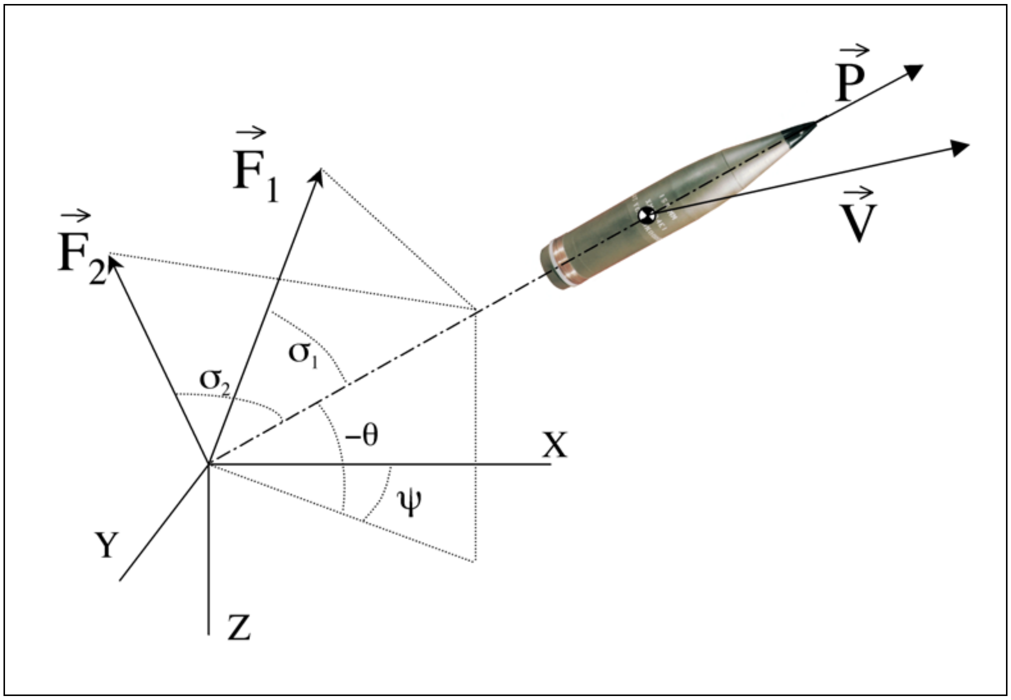

Figure 1. A spinning body within a convenient navigation coordinate system.

Let the unit vectors $\overline{P}$, $\overline{F}_1$, and $\overline{F}_2$ along $\vec{P}$, $\vec{F}_1$, and $\vec{F}_2$ be defined within the navigation (*X*, *Y*, *Z*) system as

$$\begin{aligned} \overline{P} &= (P_X, P_Y, P_Z) \\ \overline{F}_1 &= (F_{1_X}, F_{1_Y}, F_{1_Z}) \\ \overline{F}_2 &= (F_{2_X}, F_{2_Y}, F_{2_Z}) \end{aligned} \tag{1}$$

The components of $\overline{\mathrm{P}}$ are obtained from the simultaneous solution of the system

$$\begin{aligned} \overline{P} \bullet \overline{F}_1 &= \cos(\sigma_1) \\ \overline{P} \bullet \overline{F}_2 &= \cos(\sigma_2) \\ \left|\overline{P}\right| &= 1 \end{aligned} \tag{2}$$

where the field vectors ($\overline{F}_1$, $\overline{F}_2$) and the included angles (σ_1, σ_2) between $\vec{P}$ and the respective field vectors are known, estimated, or measured. This methodology is termed POINTER (U.S. Patent 6398155, June 4, 2002, Harkins & Hepner) and has been successfully employed on

telemetered data from numerous inventory and developmental projectiles instrumented with solar and magnetic sensors in flight experiments conducted to characterize projectile dynamics (Davis et al., 2004; Harkins, 2003).

3. Solving for Projectile Heading: The Legacy POINTER Solution

Angular navigation parameters are most often described in an earth-fixed Cartesian coordinate system. Given a convenient navigation coordinate system (e.g., north, east, down) and an arbitrary location L = (L_n, L_e, L_d), let the unit vector from L to the sun be $\vec{S} = (S_n, S_e, S_d)$ and the unit vector from L along the magnetic field be $\vec{M} = (M_n, M_e, M_d)$. Unfortunately, the angular variables in the system of equations 2 describing heading usually are non-separable in this coordinate system. In order to obtain the projectile heading angles, the vector along the projectile axis must be derived in a computationally convenient coordinate system and then transformed into the navigation coordinate system.

Consider a Cartesian coordinate system (A,B,C) with its origin at L, its +B axis along the solar vector, its A axis so that the local magnetic vector is in the half-plane containing the B axis and the +A axis, and its +C axis pointing in the direction that a right-hand threaded screw advances when its head is rotated from +A to +B (figure 2). This system will be referred to as the "solar" system.

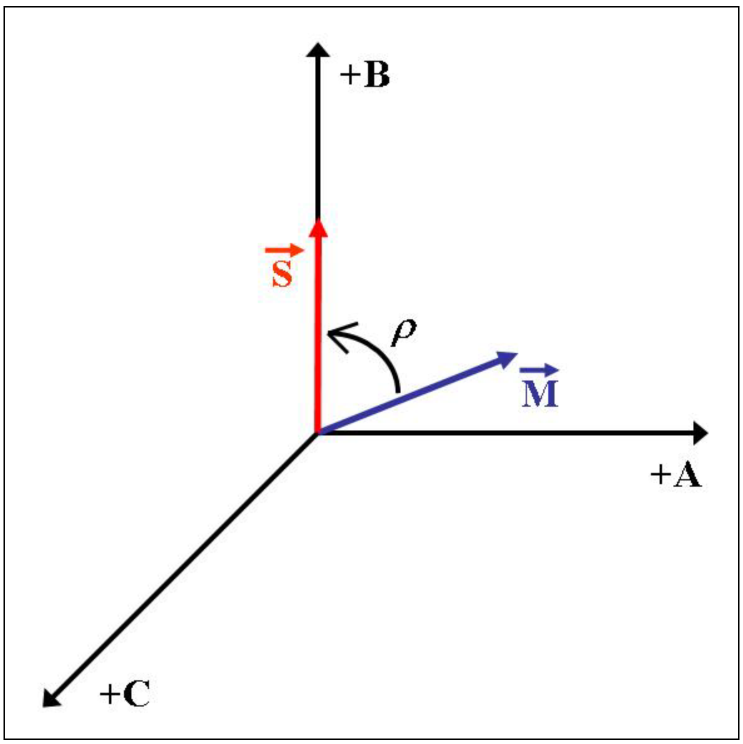

Figure 2. The "solar" coordinate system.

With the angle between $\vec{S}$ and $\vec{M}$ designated as ρ, the mappings of the axes of the solar system to the navigation system are given by

$$(0,1,0)_S \rightarrow (S_n, S_e, S_d) \tag{3}$$

$$(0,0,1)_S \rightarrow \left(\vec{M} \times \vec{S}\right)/\sin\rho \tag{4}$$

$$(1,0,0)_S \rightarrow \vec{S} \times \left[\left(\vec{M} \times \vec{S}\right)/\sin\rho\right] \tag{5}$$

where $\rho = \cos^{-1}\left(\vec{S} \bullet \vec{M}\right)$. Evaluating the cross-products in equations 4 and 5 yields expressions for the components of the respective mappings

$$\vec{M} \times \vec{S} = \left(M_e S_d - M_d S_e,\, M_d S_n - M_n S_d,\, M_n S_e - M_e S_n\right) \tag{6}$$

$$\therefore (0,0,1)_S \rightarrow \left(M_e S_d - M_d S_e,\, M_d S_n - M_n S_d,\, M_n S_e - M_e S_n\right)/\sin\rho \tag{7}$$

$$\begin{aligned} \vec{S} \times \left(\vec{M} \times \vec{S}\right) &= \vec{S} \times \left(M_e S_d - M_d S_e,\, M_d S_n - M_n S_d,\, M_n S_e - M_e S_n\right) \\ &= \begin{pmatrix} S_e\left(M_n S_e - M_e S_n\right) - S_d\left(M_d S_n - M_n S_d\right), \\ S_d\left(M_e S_v - M_d S_e\right) - S_n\left(M_n S_e - M_e S_n\right), \\ S_n\left(M_d S_n - M_n S_d\right) - S_e\left(M_e S_d - M_d S_e\right) \end{pmatrix} \end{aligned} \tag{8}$$

$$\therefore (1,0,0)_S \rightarrow \begin{pmatrix} M_n\left(S_e^2 + S_d^2\right) - S_n S_e M_e - S_n S_d M_d, \\ M_e\left(S_n^2 + S_d^2\right) - S_n S_e M_n - S_e S_d M_v, \\ M_d\left(S_n^2 + S_e^2\right) - S_n S_d M_n - S_e S_d M_e \end{pmatrix} \Bigg/ \sin\rho \tag{9}$$

Any vector defined in the "solar" system, $\vec{V}_S = \left(v_A, v_B, v_C\right)$, can be transformed to its representation in the navigation system, $\vec{V}_N = \left(v_n, v_e, v_d\right)$, by

$$\vec{V}_N = \begin{pmatrix} t_{1,1} & t_{1,2} & t_{1,3} \\ t_{2,1} & t_{2,2} & t_{2,3} \\ t_{3,1} & t_{3,2} & t_{3,3} \end{pmatrix} \vec{V}_S \tag{10}$$

where

$$
\begin{aligned}
t_{1,1} &= \left(M_n\left(S_e{}^2 + S_d{}^2\right) - S_n S_e M_e - S_n S_d M_d\right) / \sin\rho \\
t_{1,2} &= \left(M_e\left(S_n{}^2 + S_d{}^2\right) - S_n S_e M_n - S_e S_d M_d\right) / \sin\rho \\
t_{1,3} &= \left(M_d\left(S_n{}^2 + S_e{}^2\right) - S_n S_d M_n - S_e S_d M_e\right) / \sin\rho \\
t_{2,1} &= S_n \\
t_{2,2} &= S_e \\
t_{2,3} &= S_d \\
t_{3,1} &= \left(M_e S_d - M_d S_e\right) / \sin\rho \\
t_{3,2} &= \left(M_d S_n - M_n S_d\right) / \sin\rho \\
t_{3,3} &= \left(M_n S_e - M_e S_n\right) / \sin\rho
\end{aligned}
\tag{11}
$$

Given solar aspect angle (σ_S) and magnetic aspect angle (σ_M) histories derived from solarsonde and Magsonde reductions (Hepner & Harkins, 2001), these data should be interpolated onto a common time base. Then, at every time step, the components of a unit vector $(\vec{P})$ along the projectile spin axis can be readily determined in the solar system (see figure 3).

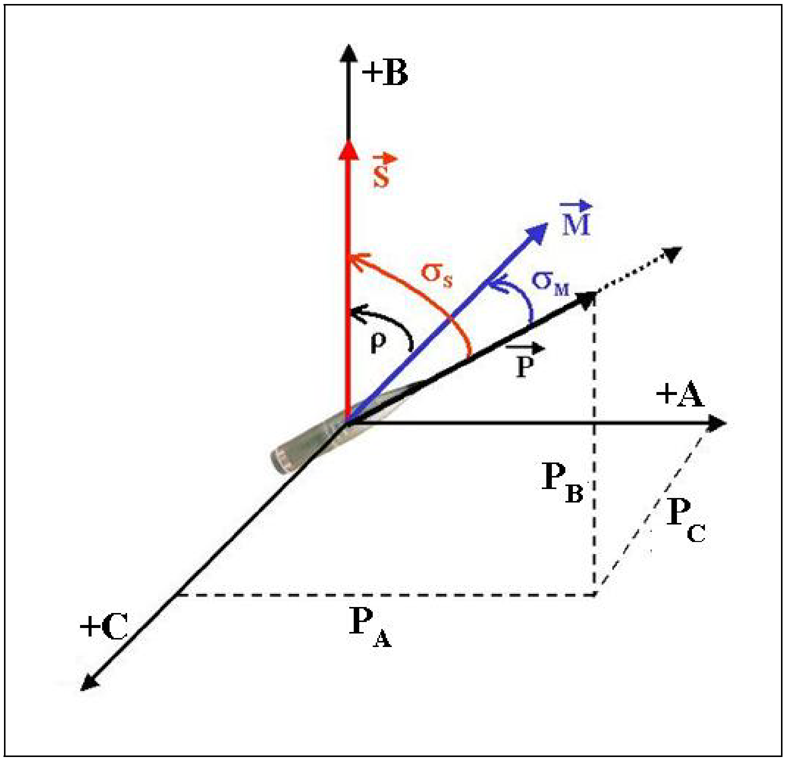

Figure 3. Components of the projectile pointing vector in the “solar” system.

$$\cos\sigma_S = (0,1,0) \bullet (P_A, P_B, P_C) = P_B \tag{12}$$

$$\cos\sigma_M = (\sin\rho, \cos\rho, 0) \bullet (P_A, P_B, P_C) = P_A \sin\rho + P_B \cos\rho \tag{13}$$

by substitution, $$P_A = \frac{\cos\sigma_M - \cos\sigma_S \cos\rho}{\sin\rho} \tag{14}$$

$$\begin{aligned} P_C &= \pm\sqrt{1 - P_A{}^2 - P_B{}^2} \\ &= \pm\sqrt{1 - \cos^2\sigma_S - \left(\frac{\cos\sigma_M - \cos\sigma_S \cos\rho}{\sin\rho}\right)^2} \end{aligned} \tag{15}$$

or $$P_C = \pm\sqrt{\sin^2\sigma_S - \left(\frac{\cos^2\sigma_M - 2\cos\sigma_M \cos\sigma_S \cos\rho + \cos^2\sigma_S \cos^2\rho}{\sin^2\rho}\right)} \tag{16}$$

The sign ambiguity of P_C is usually easily resolved with knowledge of the initial navigation orientation (e.g., launcher orientation).

Finally, with equation 10, the components in the navigation system of a unit vector along the projectile axis of rotation are given by

$$(P_n, P_e, P_d) = \begin{pmatrix} t_{1,1} & t_{1,2} & t_{1,3} \\ t_{2,1} & t_{2,2} & t_{2,3} \\ t_{3,1} & t_{3,2} & t_{3,3} \end{pmatrix} (P_A, P_B, P_C) \tag{17}$$

and the corresponding heading angles by

$$\theta = \tan^{-1}\left(\frac{P_d}{\sqrt{\left(P_n^2 + P_e^2\right)}}\right) \quad \text{where } -\pi/2 \le \theta \le \pi/2 \tag{18}$$

$$\psi = \tan^{-1}(P_e/P_n) \quad \text{where } -\pi \le \psi \le \pi \tag{19}$$

4. Coordinate Transformations With the Use of Euler Angles

Formulation of the inertial navigation problem for gun- and tube-launched projectiles requires the use of multiple coordinate systems. Trajectory time histories are best described in an earth-fixed coordinate system with its origin at the launcher. Of necessity, strap-down sensor measurements are made in a flight-body-fixed coordinate system, and target locations are most naturally described in another earth-fixed system.

The first coordinate system is right-handed Cartesian (*I*, *J*, *K*) with its origin at the launch site. This will be referred to as the “earth-fixed” system and the axes are defined by

- The *I* and *J* axes, which define a plane tangential to the earth’s surface at the origin.

- The K axis, which is perpendicular to the earth's surface with positive downward, i.e., in the direction of gravity.
- The I axis, which is chosen so that the centerline of the launcher is in the I-K plane.

Down-range travel is then measured along the I axis, deflection along the J axis (positive to the right when one is looking down range), and altitude along the K axis (positive downward) (see figure 4).

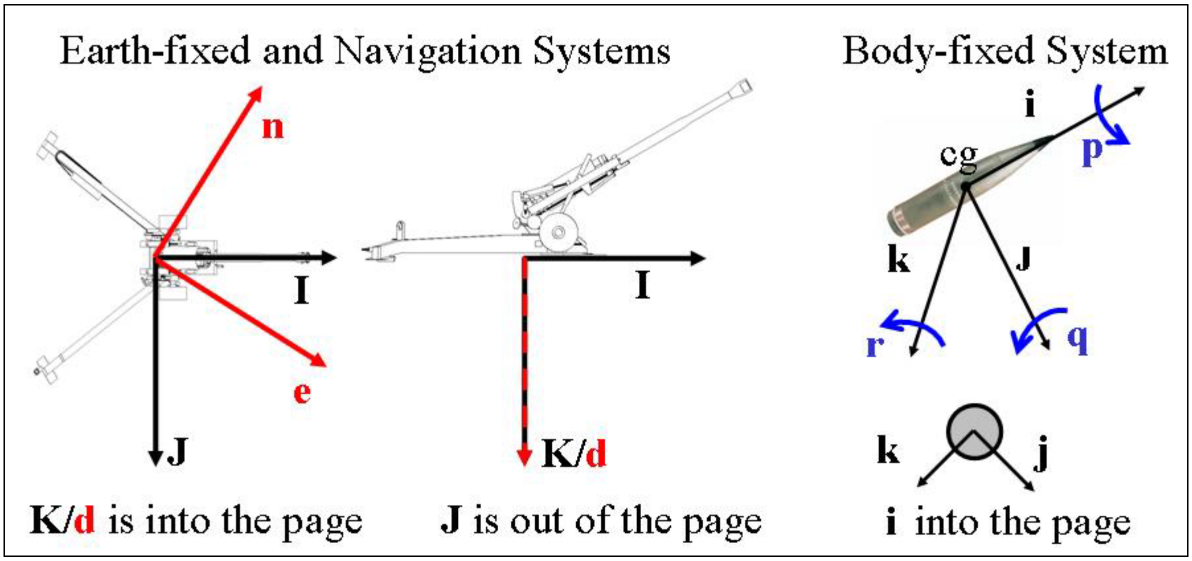

Figure 4. Coordinate systems.

The second system is convenient for aeroballistic computations of rigid projectiles' flights and for describing the locations and orientations of such projectiles' components. This system is right-handed Cartesian (i, j, k) with its origin at the center of gravity (c.g.) of the flight body. For rotating flight bodies, the projectile-fixed coordinate system usually has its i axis along the projectile axis of symmetry, i.e., the spin axis (with positive in the direction of travel at launch). The j and k axes are then oriented so as to complete the right-handed orthogonal system (figure 4). Spin (p), pitch (q), and yaw (r) rates are measured about these axes. This will be referred to as the "body-fixed" system.

The third coordinate system (n,e,d) is commonly employed to specify locations on or near the earth's surface, i.e., north, east, and down. This will be referred to as the "navigation" system.

The earth-fixed and body-fixed coordinate systems are related through an Euler rotation sequence beginning with a rotation of the earth-fixed frame about the K axis through the yaw angle ψ. The system is then rotated about the new J' axis through the pitch angle θ. Finally, the system is rotated about the i axis through the roll angle ϕ. The two systems are related by the direction cosine matrix (DCM), T_{Eb}, with the subscript denoting earth fixed to body fixed. This transfor-mation matrix is

$$T_{Eb} = \begin{pmatrix} c_\psi c_\theta & s_\psi c_\theta & -s_\theta \\ c_\psi s_\theta s_\phi - s_\psi c_\phi & s_\psi s_\theta s_\phi + c_\psi c_\phi & c_\theta s_\phi \\ c_\psi s_\theta c_\phi + s_\psi s_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi & c_\theta c_\phi \end{pmatrix}, \tag{20}$$

where $c_\bullet$ is $\cos(\bullet)$, and $s_\bullet$ is $\sin(\bullet)$. Figure 5 shows both the earth-fixed and the body-fixed coordinate systems and the Euler angle relations between them. Transformations between any two right-handed Cartesian systems can be similarly defined with appropriate values for the Euler angles.

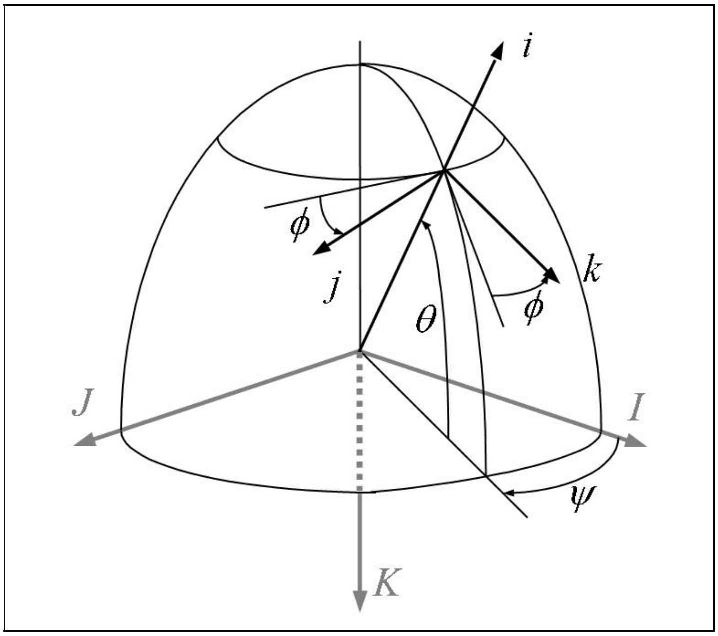

Figure 5. Earth-fixed and body-fixed systems and the Euler angle rotations.

5. Solving for Projectile Heading: A Unique POINTER Solution

In section 3, it was shown that the angular variables describing projectile heading can be made independent and therefore separable in an appropriate coordinate system. In section 4, it was shown that transformation between two similar Cartesian systems is described by the Euler rotation sequence. With the choice of a suitable coordinate system, these two characteristics are combined to achieve a system of equations whose simultaneous solution is unique. This "computational" coordinate system is somewhat similar to the "solar" system of section 3 and is shown next. This coordinate system (I_C, J_C, K_C) has its origin at the projectile c.g. It is oriented so that the unit vector to the sun $(\vec{S})$ is located along the $-K_C$ axis; a unit vector $(\vec{M})$ along the local magnetic vector is in the half-plane containing the K_C axis and the $+I_C$ axis, and its $+J_C$ axis is pointing in the direction that a right-hand threaded screw advances when its head is rotated from $+K_C$ to $+I_C$ (figure 6).

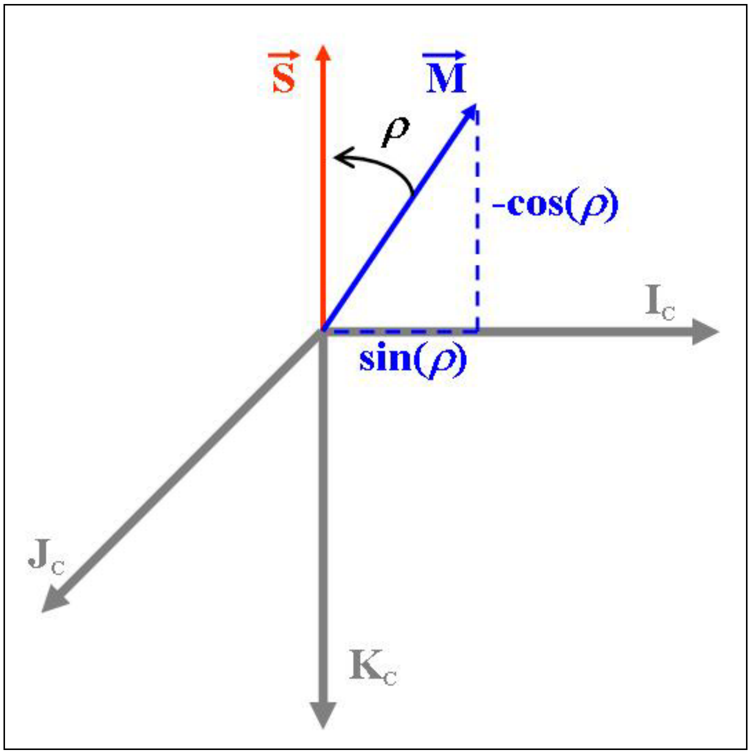

Figure 6. The "computational" coordinate system.

With the angle between $\vec{S}$ and $\vec{M}$ designated as ρ, the mappings of the axes of the computational system to the navigation system are given by

$$(0,0,-1)_C \rightarrow (S_n, S_e, S_d) \tag{21}$$

$$(0,1,0)_C \rightarrow \left(\vec{M} \times \vec{S}\right) / \sin\rho \tag{22}$$

$$(1,0,0)_C \rightarrow \vec{S} \times \left(\left(\vec{M} \times \vec{S}\right) / \sin\rho \right) \tag{23}$$

where $\rho = \cos^{-1}\left(\vec{S} \bullet \vec{M}\right)$. Proceeding as in section 3, the components of the respective mappings are

$$(0,0,1)_C \rightarrow (-S_n, -S_e, -S_d) \tag{24}$$

$$(0,1,0)_C \rightarrow \left(M_e S_d - M_v S_d,\ M_v S_d - M_n S_d,\ M_n S_d - M_e S_d\right) / \sin\rho \tag{25}$$

$$(1,0,0)_C \rightarrow \left. \begin{pmatrix} M_n\left(S_e^2 + S_d^2\right) - S_n S_e M_e - S_n S_d M_v, \\ M_e\left(S_n^2 + S_d^2\right) - S_n S_e M_n - S_e S_d M_v, \\ M_v\left(S_n^2 + S_e^2\right) - S_n S_d M_n - S_e S_d M_e \end{pmatrix} \right/ \sin\rho \tag{26}$$

Thus, any vector defined in the "computational" system, $\vec{V}_C = \left(v_{I_C}, v_{J_C}, v_{K_C}\right)$, can be transformed to its representation in the navigation system, $\vec{V}_N = \left(v_n, v_e, v_d\right)$, by

$$\vec{V}_N = T_{CN}\,\vec{V}_C = \begin{pmatrix} t_{1,1} & t_{1,2} & t_{1,3} \\ t_{2,1} & t_{2,2} & t_{2,3} \\ t_{3,1} & t_{3,2} & t_{3,3} \end{pmatrix} \vec{V}_C \tag{27}$$

where

$$\begin{aligned}
t_{1,1} &= \left(M_n\left(S_e{}^2 + S_d{}^2\right) - S_n S_e M_e - S_n S_d M_v\right) \Big/ \sin\rho \\
t_{1,2} &= \left(M_e\left(S_n{}^2 + S_d{}^2\right) - S_n S_e M_n - S_e S_d M_v\right) \Big/ \sin\rho \\
t_{1,3} &= \left(M_d\left(S_n{}^2 + S_e{}^2\right) - S_n S_d M_n - S_e S_d M_e\right) \Big/ \sin\rho \\
t_{2,1} &= \left(M_e S_d - M_d S_e\right) / \sin\rho \\
t_{2,2} &= \left(M_d S_n - M_n S_d\right) / \sin\rho \\
t_{2,3} &= \left(M_n S_e - M_e S_n\right) / \sin\rho \\
t_{3,1} &= -S_n \\
t_{3,2} &= -S_e \\
t_{3,3} &= -S_d
\end{aligned} \tag{28}$$

Continuing as before, solar aspect angle (σ_S) and magnetic aspect angle (σ_M) histories derived from solarsonde and Magsonde reductions should be interpolated onto a common time base. Then, at every time step, the components of a unit vector $(\vec{P}_C)$ along the projectile spin axis can be determined in the computational system (see figure 7).

As before, equations for the solar and magnetic aspect angles lead to solutions for two of the pointing vector components.

$$\cos\sigma_S = (0,0,-1) \bullet \left(P_{I_C}, P_{J_C}, P_{K_C}\right) = -P_{K_C} \tag{29}$$

$$\cos\sigma_M = (\sin\rho, 0, -\cos\rho) \bullet \left(P_{I_C}, P_{J_C}, P_{K_C}\right) = P_{I_C}\sin\rho - P_{K_C}\cos\rho \tag{30}$$

by substitution,
$$P_{I_C} = \frac{\cos\sigma_M - \cos\sigma_S \cos\rho}{\sin\rho} \tag{31}$$

A solution for the third component follows from the equations for the Euler sequence necessary for transformation from the computational system to the body-fixed system as shown in figure 8. The transformation matrix from the computational system to the body-fixed system is given by equation 20, mutatis mutandis.

$$T_{Cb} = \begin{pmatrix} c_{\psi_{Cb}} c_{\theta_{Cb}} & s_{\psi_{Cb}} c_{\theta_{Cb}} & -s_{\theta_{Cb}} \\ c_{\psi_{Cb}} s_{\theta_{Cb}} s_{\phi_{Cb}} - s_{\psi_{Cb}} c_{\phi_{Cb}} & s_{\psi_{Cb}} s_{\theta_{Cb}} s_{\phi_{Cb}} + c_{\psi_{Cb}} c_{\phi_{Cb}} & c_{\theta_{Cb}} s_{\phi_{Cb}} \\ c_{\psi_{Cb}} s_{\theta_{Cb}} c_{\phi_{Cb}} + s_{\psi_{Cb}} s_{\phi_{Cb}} & s_{\psi_{Cb}} s_{\theta_{Cb}} c_{\phi_{Cb}} - c_{\psi_{Cb}} s_{\phi_{Cb}} & c_{\theta_{Cb}} c_{\phi_{Cb}} \end{pmatrix} \tag{32}$$

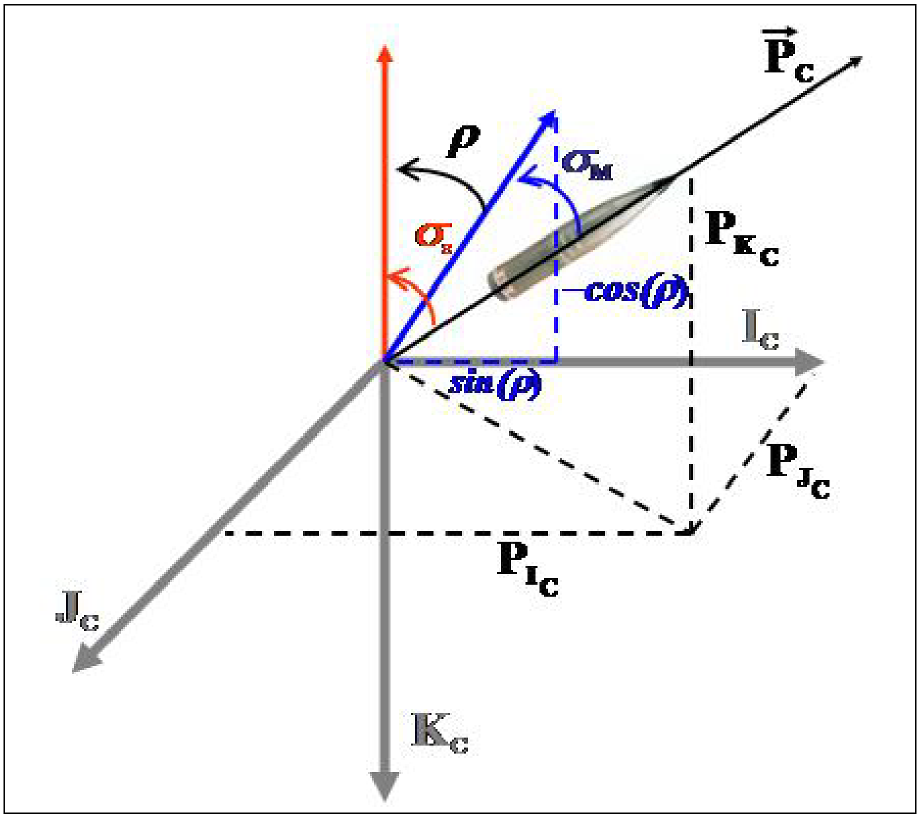

Figure 7. Components of the projectile pointing vector in the “computational” system.

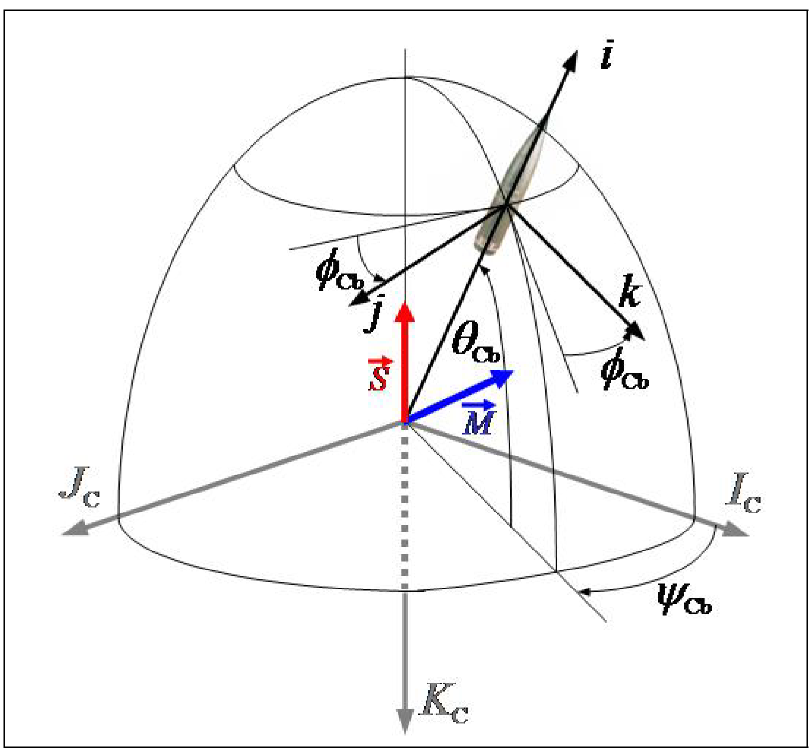

Figure 8. Computational and body-fixed systems and the Euler angle rotations.

In particular, the unit solar vector, $\vec{S}$, and the magnetic field vector, $\vec{m}_C = \vec{M}\left(|\vec{m}_C|\right)$, are mapped, respectively, into the body-fixed system by

$$\begin{pmatrix} S_i \\ S_j \\ S_k \end{pmatrix} = \begin{pmatrix} c_{\psi_{Cb}} c_{\theta_{Cb}} & s_{\psi_{Cb}} c_{\theta_{Cb}} & -s_{\theta_{Cb}} \\ c_{\psi_{Cb}} s_{\theta_{Cb}} s_{\phi_{Cb}} - s_{\psi_{Cb}} c_{\phi_{Cb}} & s_{\psi_{Cb}} s_{\theta_{Cb}} s_{\phi_{Cb}} + c_{\psi_{Cb}} c_{\phi_{Cb}} & c_{\theta_{Cb}} s_{\phi_{Cb}} \\ c_{\psi_{Cb}} s_{\theta_{Cb}} c_{\phi_{Cb}} + s_{\psi_{Cb}} s_{\phi_{Cb}} & s_{\psi_{Cb}} s_{\theta_{Cb}} c_{\phi_{Cb}} - c_{\psi_{Cb}} s_{\phi_{Cb}} & c_{\theta_{Cb}} c_{\phi_{Cb}} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \tag{33}$$

$$\begin{pmatrix} m_i \\ m_j \\ m_k \end{pmatrix} = \begin{pmatrix} c_{\psi_{Cb}} c_{\theta_{Cb}} & s_{\psi_{Cb}} c_{\theta_{Cb}} & -s_{\theta_{Cb}} \\ c_{\psi_{Cb}} s_{\theta_{Cb}} s_{\phi_{Cb}} - s_{\psi_{Cb}} c_{\phi_{Cb}} & s_{\psi_{Cb}} s_{\theta_{Cb}} s_{\phi_{Cb}} + c_{\psi_{Cb}} c_{\phi_{Cb}} & c_{\theta_{Cb}} s_{\phi_{Cb}} \\ c_{\psi_{Cb}} s_{\theta_{Cb}} c_{\phi_{Cb}} + s_{\psi_{Cb}} s_{\phi_{Cb}} & s_{\psi_{Cb}} s_{\theta_{Cb}} c_{\phi_{Cb}} - c_{\psi_{Cb}} s_{\phi_{Cb}} & c_{\theta_{Cb}} c_{\phi_{Cb}} \end{pmatrix} \begin{pmatrix} \sin(\rho) \\ 0 \\ -\cos(\rho) \end{pmatrix} \left(|\vec{m}_C|\right) \tag{34}$$

By inspection of figures 7 and 8, it is seen that $\theta_{Cb} = 90 - \sigma_S$. For a calibrated tri-axial magnetometer whose axes are parallel to the body-fixed system, equation 34 gives the output of the respective axes. The spin-axis-aligned magnetometer's output is

$$m_i = c_{\psi_{Cb}} c_{\theta_{Cb}} \sin(\rho)\left(|\vec{m}_C|\right) + s_{\theta_{Cb}} \cos(\rho)\left(|\vec{m}_C|\right) \tag{35}$$

Rearranging terms yields

$$\psi_{Cb} = \cos^{-1}\left(\frac{m_i - s_{\theta_{Cb}} \cos(\rho)\left(|\vec{m}_C|\right)}{c_{\theta_{Cb}} \sin(\rho)\left(|\vec{m}_C|\right)}\right) \tag{36}$$

Because $\cos(\psi) = \cos(-\psi)$, the correct polarity of ψ_{Cb} must be determined before we can proceed. Since the optical detectors in the solar measurement system are essentially binary devices that have zero output when not aligned with the sun and unit output when aligned, continuous values for S_j and S_k are not available[1]. However, any optical detector whose field of view is coplanar with the projectile spin axis provides a roll angle (ϕ_{Cb}) index when aligned with the sun. Typically, there are two such detectors oriented along the $+k$ and $-k$ body axes. If we consider the $+k$-oriented detector when it is aligned with the sun, it follows from equation 33 that $\phi_{Cb} = 180°$. With equation 34 for the corresponding values of either m_j or m_k at this time, the correct polarity of ψ_{Cb} is identified. After this polarity has been determined, it needs only to be rechecked if and when $|\cos(\psi_{Cb})| \to 1$.

With the correct value of ψ_{Cb} in hand and observing from figures 7 and 8 that $\tan(\psi_{Cb}) = P_{J_C} / P_{I_C}$, we compute

$$P_{J_C} = \tan(\psi_{Cb})\, P_{I_C} \tag{37}$$

Finally, with equations 27 and 28, the pointing vector in the navigation system is uniquely given by

$$\vec{P}_N = T_{CN}\ \vec{P}_C \tag{38}$$

[1]If continuous tri-axial measurements with respect to two earth-fixed fields are available, the possibility of computing Euler angles in multiple ways using different combinations of the sensor output can be exploited to identify and correct for individual sensor corruption and/or failure (Jagadish & Chang 2007).

Elevation and azimuth in the navigation system are then computed with equations 18 and 19, respectively.

6. Summary

A system of equations relating strap-down optical and magnetic sensor output and projectile heading have been identified whose simultaneous solution uniquely identifies the projectile azimuth and elevation. This represents an improvement of an earlier formulation that led to an ambiguous solution. With a unique solution available, the possibility of on-board and automated computation of projectile heading is opened. This should be attempted as soon as practicable for test and evaluation purposes.

For the future, there are a number of potential alternatives to the solar field measurements discussed herein, which when successfully realized, will enable a tactical implementation of a POINTER system using low-cost sensors.

7. References

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